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Applied Electronics

A First Course in Electronics,
Electron Tubes, and Associated Circuitry.

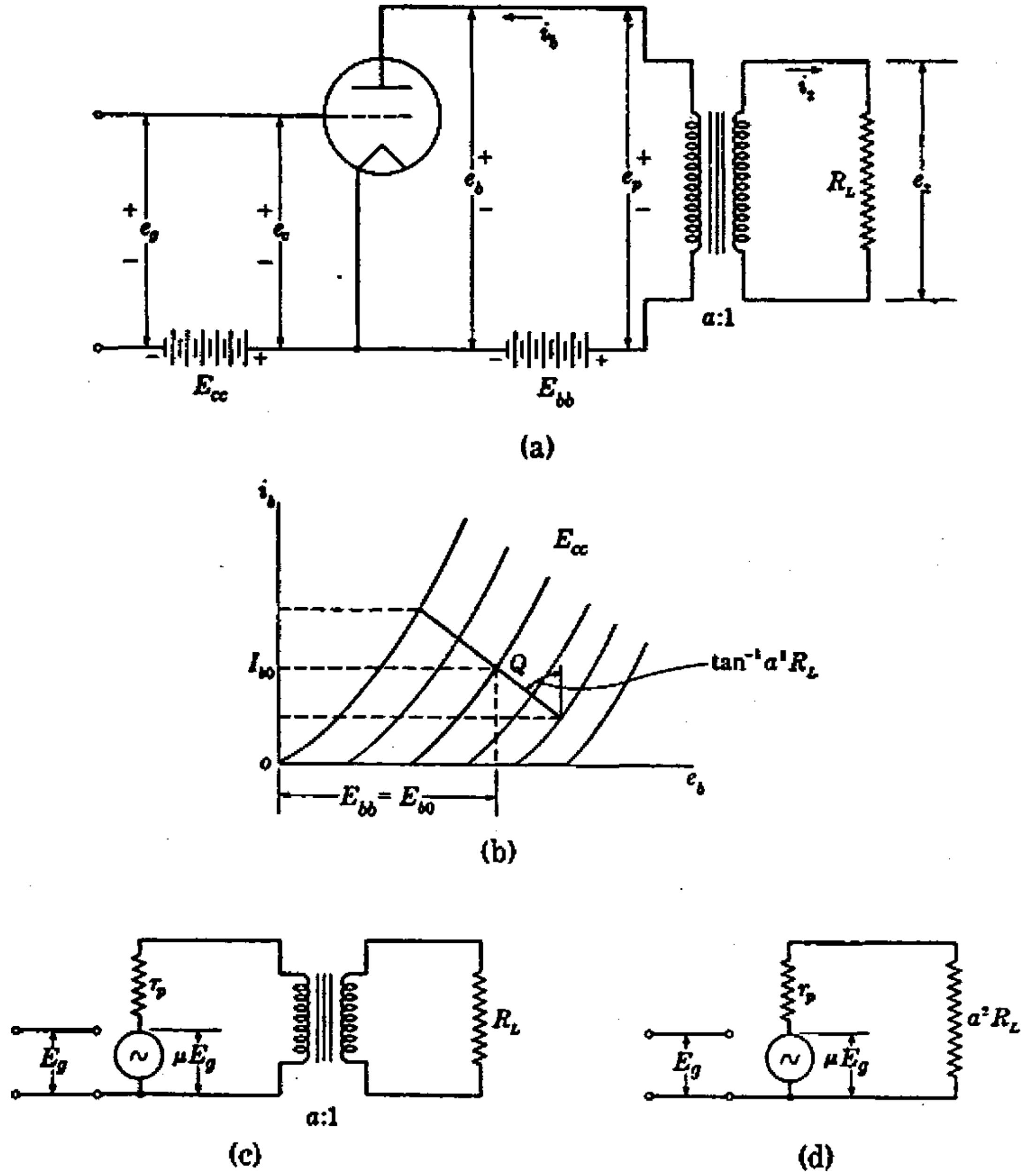
By the Members of the Staff of the
Department of Electrical Engineering
Massachusetts Institute of Technology.

12. USE OF AN IDEAL OUTPUT TRANSFORMER FOR IMPEDANCE MATCHING

Advantages of adjustment of the load resistance to suit the tube are described in the preceding articles. In practice, however, an arbitrary choice of the load resistance to realize these advantages is usually not feasible because, for example, the load may be a device already available, or one whose design involves inherent limitations of resistance. Hence, in amplifiers, output transformers are generally used between the tube and the load. The characteristics of such transformers are described in the volume on magnetic circuits and transformers. From the considerations of Art. 11, it is apparent that a value of load resistance equal to the plate resistance is not desirable when maximum power with a prescribed amount of harmonic generation is wanted, and that a transformer ratio to cause the actual load resistance to have an apparent value in the plate circuit of about twice the plate resistance of the tube is needed when the tube is a triode and the quiescent plate voltage is specified.

The circuit diagram for an amplifier with an output transformer of turns ratio a and a resistance load, is shown in Fig. 28a. If the transformer is assumed to be ideal, the path of operation on the plate characteristics is as shown in Fig. 28b. Because the ideal transformer has no losses, the windings have no resistance, and the quiescent operating point Q has an abscissa E_{b0} that equals E_{bb}. In an actual transformer, the quiescent point lies somewhat to the left of this abscissa by the amount of the direct voltage drop in the primary winding; that is, on a line through point $(E_{bb}, 0)$ with a slope of $-(1/R_{dc})$, where R_{dc} is the direct-current resistance of the winding in series with the plate battery (*see* Art. 4, Ch. IX, for a somewhat similar condition in the resistance-capacitance-coupled amplifier). The path of the operating point on Fig. 28b is along a load line having a slope $-(1/a^2R_L)$, since the apparent impedance as viewed from the tube into the transformer is a^2R_L. Thus the resistances used for the determination of the direct-current and alternating-current conditions on the plate characteristics are different.

If the path of operation remains in the linear region of the plate characteristics, the equivalent circuit for alternating components is that shown in Fig. 28c, which may be further simplified to that of Fig. 28d because of the impedance transformation property of the transformer. For maximum power output with a prescribed amount of harmonic generation and a prescribed quiescent plate voltage, the considerations of Art. 11 show that the turns ratio of the transformer should be



(a)

(b)

(c)

(d)

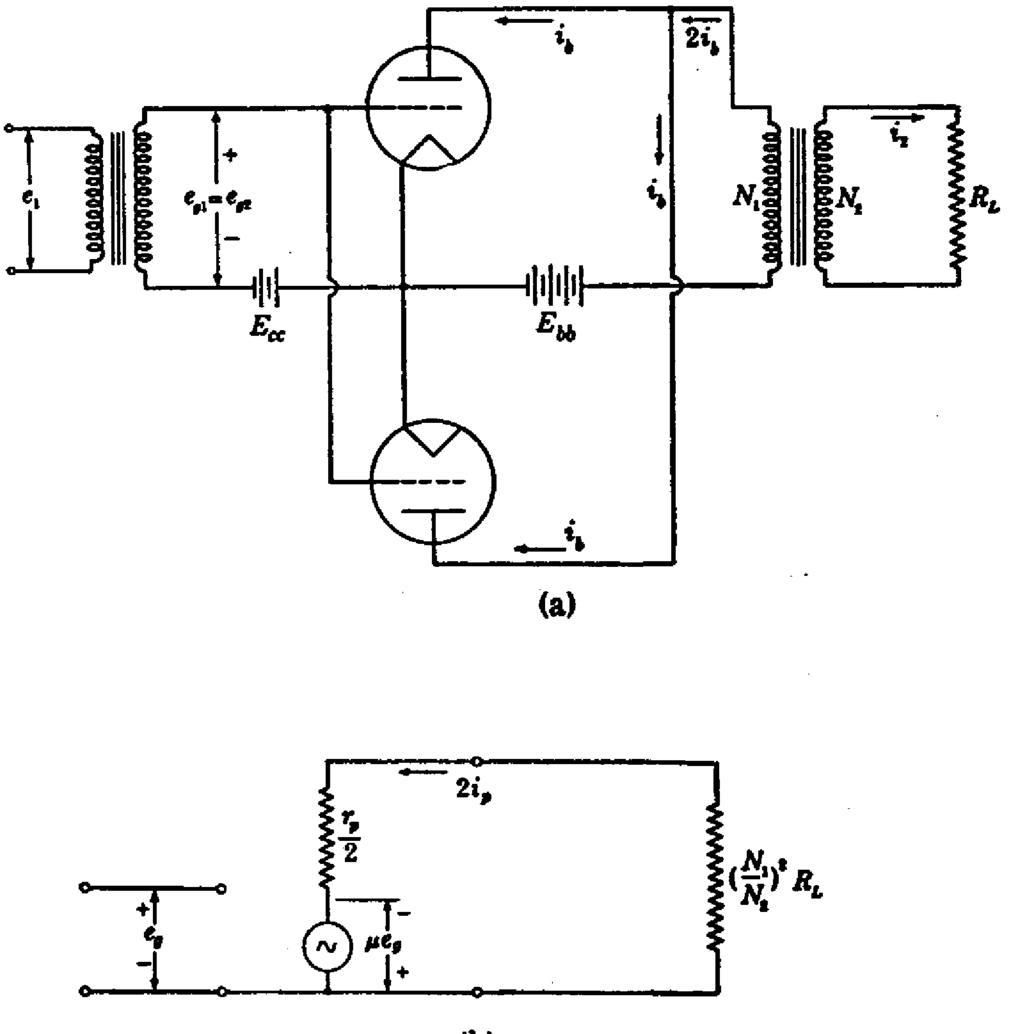
FIG. 28. Linear Class A_1 triode operation with an ideal output transformer.

$$a = \sqrt{\frac{2r_p}{R_L}}. \qquad [189]$$

The output transformer serves a threefold purpose; namely, (a) it makes possible the realization of the conditions for maximum power output with almost any tube and load, since its turns ratio can be selected; (b) it eliminates $I_b^2R_L$, the direct-current component of power in the load, because I_b is confined to the low-resistance primary winding of the transformer and does not exist in R_L; and (c) it serves as an electrical isolator between the tube and the load circuit, since with it the potential of any one point in the load circuit may be given any desired value without regard to potentials of points in the tube circuit.

13. Parallel operation; Class A_1

One method of obtaining a power output greater than that obtainable from a single tube is the use of two tubes connected in parallel. Another method, which has some advantages over the parallel connection, is discussed in Art. 14.



(a)

(b)

Fig. 29. Circuit diagram and equivalent circuit for two identical triodes connected in parallel.

When two identical tubes are connected in parallel, the total plate-circuit current for particular values of the interelectrode voltages is double that of either tube. The circuit diagram is that shown in Fig. 29a. The equivalent circuit for varying components of current and voltage when the operation is restricted to the linear region of the plate characteristics is that shown in Fig. 29b. The equivalent tube that gives the same performance as the two tubes in parallel is one with a plate resistance equal to one-half the plate resistance of either tube and a plate current double that of either tube. The ideal output transformer reflects the load resistance R_L into the plate circuit of the tubes as the value $(N_1/N_2)^2R_L$.

The operating points for the two tubes move simultaneously along the same path in the same direction on the plate characteristics. The analysis of Art. 11 then applies to the equivalent tube exactly as it does to either of the actual tubes. For a sinusoidal grid-signal voltage e_g, the harmonic generation caused by nonlinearity in the tubes comprises both odd and even harmonics of the grid-signal-voltage frequency in the total plate-circuit current and the plate voltage.

The maximum power output with a prescribed amount of harmonic generation and a prescribed quiescent plate voltage for the two tubes is twice that for a single tube, and the load resistance that corresponds to this output is one that is about twice the plate resistance of the equivalent tube. Thus, for maximum power output with a prescribed amount of harmonic generation and a prescribed quiescent plate voltage,

$$2\left(\frac{r_p}{2}\right) = \left(\frac{N_1}{N_2}\right)^2 R_L, \qquad [190]$$

and the transformer ratio is

$$\frac{N_2}{N_1} = \sqrt{\frac{R_L}{r_p}}. \qquad [191]$$

14. PUSH-PULL OPERATION; CLASS A_1

A method of connecting two tubes that is often preferable to the parallel connection discussed in the previous article when increased power output is desired is shown in Fig. 30. The tubes are connected so that the plate current in one tube decreases when that in the other tube increases. This type of connection is commonly called the *push-pull* connection. The push-pull connection has numerous advantages over the parallel connection, one of the most important being the elimination of even-harmonic generation. As a result, the maximum power output with a prescribed amount of harmonic generation is greater than that from two tubes in parallel, and the push-pull circuit is extensively used not only for Class A operation but also for Class B and Class C operation.

It is assumed in the following analysis of the push-pull amplifier that: First, the operation is restricted to the negative-grid region of the tube characteristics and consequently the grid current is considered to be negligible; second, the transformers are ideal; third, the load is a pure resistance; and, fourth, the tubes have identical characteristics. Primed symbols are used to distinguish the quantities in one tube from those in the other, but the arbitrary assignments of direction and polarity are made symmetrically with respect to the common-cathode point on the diagram. The effects of nonlinearity on the alternating-current operation are neglected at first; later they are taken into account.

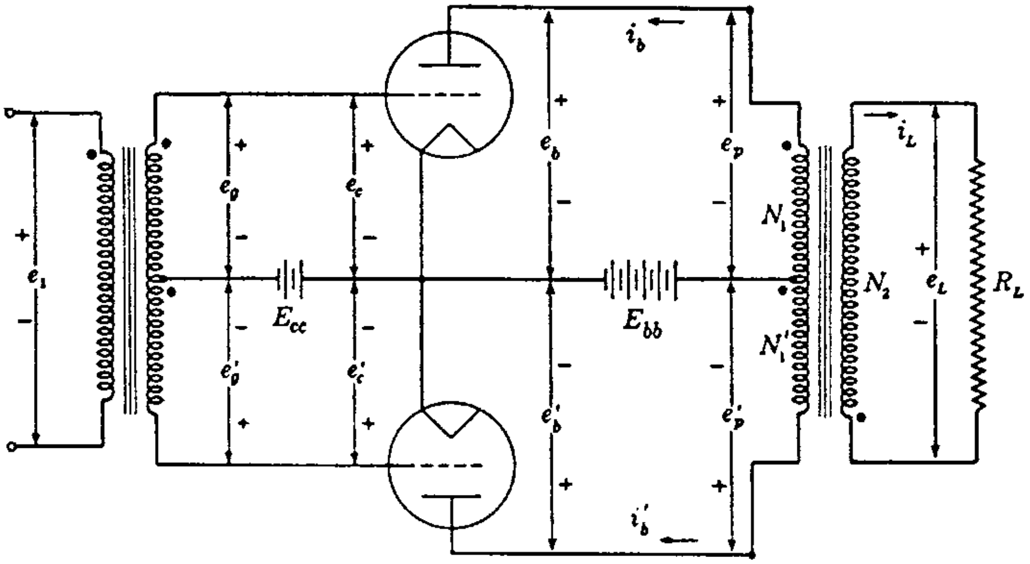


FIG. 30. Push-pull connection of two triodes.

14a. *Determination of Quiescent Operating Point.* — The determination of the quiescent operating point Q on the plate characteristics is made in the same manner as for a single tube in Art. 12, Fig. 28b. Thus when

$$e_1 = 0, \tag{192}$$

$$e_g = e'_g = 0, \tag{193}$$

and

$$e_c = e'_c = E_{cc}. \tag{194}$$

Also

$$e_p = e'_p = 0 \tag{195}$$

and

$$e_b = e'_b = E_{b0} = E_{bb}. \tag{196}$$

The quiescent plate currents I_{b0} and I'_{b0} are therefore equal in the two tubes and correspond to the point on the plate characteristics at the voltage co-ordinates given by Eqs. 194 and 196. The total quiescent plate current through the plate-power supply or battery is thus twice the current for one tube, and, as far as the quiescent operating conditions are concerned, the tubes operate *in parallel.*

The winding directions in the output transformer with its center-tapped primary winding are shown in Fig. 32. The dots used to indicate polarity have the significance that a rate of change of flux in the core that makes one of the dot-marked coil ends instantaneously positive with respect to the corresponding unmarked end also makes the other two dot-marked coil ends instantaneously positive with respect to their respective unmarked ends. For the zero-grid-signal condition

$$i_b = i_b' = I_{b0}, \tag{197}$$

the magnetomotive force in the upper winding of N_1 turns tends to send flux in a counterclockwise direction in the core, and that in the lower winding of N_1 turns tends to send flux in the clockwise direction. The net magnetization of the core resulting from the quiescent components of the plate currents is therefore zero. This cancellation of magnetization is one of the principal advantages of the push-pull connection over the single-tube or parallel connection. For a given power output and amount of harmonic generation caused by nonlinearity in the transformer, the transformer in a push-pull amplifier may be lighter in weight and less expensive than the one in a parallel-tube amplifier, because it is not necessary to provide a large core with an air gap to prevent the magnetic saturation and resulting waveform distortion caused by the average component of plate current. The effect of a direct current superposed on the alternating current in an iron-cored coil is discussed in the volume on magnetic circuits and transformers.

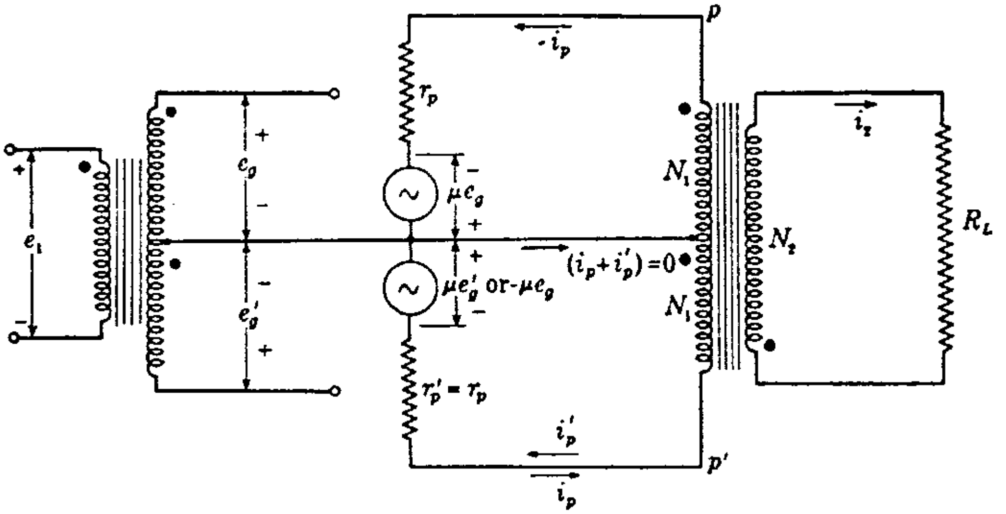


FIG. 31. Equivalent circuit for identical tubes with the varying components of current and voltage restricted to the linear region of the tube characteristics.

14b. *Operation in a Linear Region.* — For a small grid-signal voltage when the path of the operating point is *restricted to the linear region of the tube characteristics,* the operation for varying components of current and voltage may be obtained through use of the equivalent circuit for the tube. The total plate current and voltage are then obtainable through superposition of the quiescent and varying components. The equivalent circuit for varying components is that shown in Fig. 31.

If the input transformer is wound in a manner similar to the output transformer of Fig. 32, with equal numbers of turns on the two halves of the center-tapped secondary winding, then

$$e_g' = -e_g. \qquad [198]$$

Because of the linearity and symmetry of the circuit, it follows that

$$i_p' = -i_p. \qquad [199]$$

Thus the total current through the plate-power supply or battery is

$$i_b + i_b' = I_{b0} + i_p + I_{b0} - i_p \qquad [200]$$

$$= 2I_{b0} = \text{constant} \qquad [201]$$

and contains no varying component. For this reason, it is not important that the plate-power supply have low internal impedance when operation is restricted to the linear region of the curves. Also, if a self-bias resistor is used (*see* Arts. 4 and 11, Ch. IX), no by-pass capacitor is required to prevent fluctuations in plate current from affecting the grid voltage during strictly linear operation.

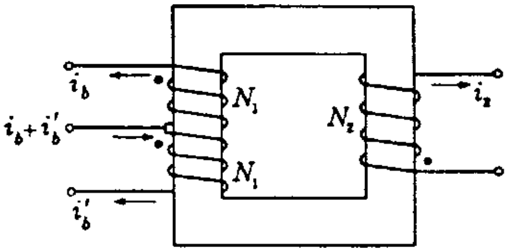


FIG. 32. Winding directions in the output transformer.

The ideal output transformer and load introduce an apparent resistance equal to $4(N_1/N_2)^2R_L$ between points p and p' in the equivalent circuit of Fig. 31 because of the impedance-transformation property of the transformer. The equivalent circuit including this apparent resistance is thus that of Fig. 33a, where the center connection shown dotted is unnecessary, because no varying component of current exists in it. From this circuit, the apparent load resistance for each tube is seen to be $2(N_1/N_2)^2R_L$, or half the *plate-to-plate resistance* R_{pp}, where

$$R_{pp} = 4\left(\frac{N_1}{N_2}\right)^2 R_L. \qquad ▶[202]$$

If one tube were removed from its socket in the circuit of Fig. 30, the second tube would then have an apparent load resistance of $(N_1/N_2)^2R_L$. Re-insertion of the first tube would therefore double the effective load resistance of the second tube. This reaction of one circuit on the other would not occur if the output transformer were eliminated and a center-tapped load resistor were used. Thus it may be concluded that the effect is associated with the autotransformer effect or coupling between the two plate circuits by the output transformer.

This analysis indicates that for small grid-signal voltages the load line for the path of operation of one tube on the plate characteristics in the linear region passes through the quiescent operating point, as shown in Fig. 28, but has a slope of

$$-\frac{1}{2\left(\frac{N_1}{N_2}\right)^2 R_L} \text{ instead of } -\frac{1}{\left(\frac{N_1}{N_2}\right)^2 R_L}$$

However, since the analysis here is restricted to operation in the linear region of the tube characteristics, it must not be inferred that the maximum power output as well as the corresponding optimum load resistance and harmonic generation can be obtained from this load line by the methods of Arts. 8 and 11. The reaction of the second tube through the transformer affects the path of operation in the first, and it is shown subsequently that over an extended region the path of operation is *not* a straight line on the plate characteristics of either tube, even though R_L is a pure resistance and the output transformer is ideal.

The circuit of Fig. 33a reduces to that of Fig. 33b. This diagram shows that as far as varying components are concerned the two tubes are in series but, as was previously stated, they are in parallel as far as quiescent components are concerned.

However, the foregoing is not the only possible point of view. The circuit of Fig. 33c is equivalent to Figs. 33a and 33b for power considerations alone; it is not equivalent for the voltage and current at the load resistor. Also, Fig. 33c is the equivalent circuit including the apparent resistance offered by the transformer and load in Fig. 33d. Figure

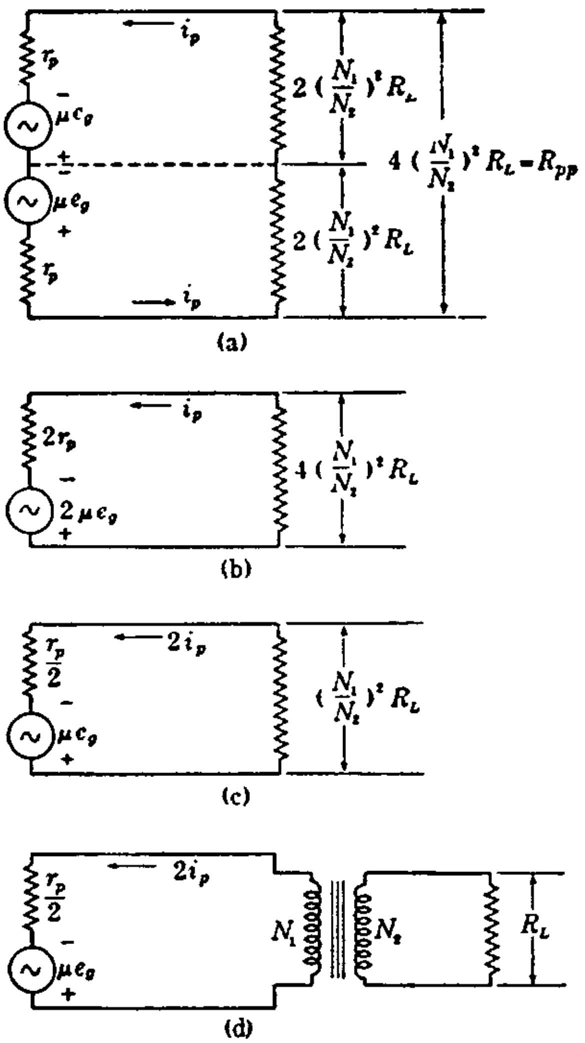


FIG. 33. Equivalent circuits for the linear Class A_1 push-pull amplifier.

33c is identical with Fig. 29b; thus it may be stated that the push-pull connection is equivalent, as far as varying components and power considerations are concerned, to two parallel-connected tubes operating into one-half the center-tapped winding of the output transformer. Either of the foregoing alternative concepts may be useful, but they are applicable only to operation over the linear region of the tube's characteristic curves.

14c. *Operation over a Range Extending beyond the Linear Region.*—When the path of operation extends beyond the linear region of the plate characteristics, harmonics are generated in the plate current that are not present in the grid-signal voltage. For example, if the grid-signal voltage is sinusoidal and expressible as

$$e_g = \sqrt{2}E_g \cos \omega t, \tag{203}$$

the plate current under conditions of no harmonic generation is

$$i_b = I_{b0} + \sqrt{2}I_p \cos \omega t; \tag{204}$$

but with harmonic generation it is expressible as the Fourier series,

$$i_b = I_{b0} + I_{p0} + \sqrt{2}(I_{p1} \cos \omega t + I_{p2} \cos 2\omega t + I_{p3} \cos 3\omega t + \cdots), \tag{205}$$

as is demonstrated in Art. 8. Because of the symmetry in the push-pull circuit, the plate current in the second tube is similar to Eq. 205, but with ωt replaced by $\omega t + 180°$. Thus

$$\begin{aligned} i_b' = {} & I_{b0} + I_{p0} + \sqrt{2}[I_{p1} \cos (\omega t + 180°) + I_{p2} \cos (2\omega t + 360°) \\ & + I_{p3} \cos (3\omega t + 540°) + \cdots] & [206] \\ = {} & I_{b0} + I_{p0} + \sqrt{2}[-I_{p1} \cos \omega t + I_{p2} \cos 2\omega t - I_{p3} \cos 3\omega t \\ & + \cdots]. & [207] \end{aligned}$$

One feature assumed in an ideal transformer is that the exciting current is negligible, which is equivalent to the statement that the magnetomotive force required to magnetize the core is zero. Consequently, the sum of the magnetomotive forces caused by currents in the windings is zero in a given direction around the core; and in a two-winding ideal transformer the current ratio is the inverse of the turns ratio. In the ideal push-pull output transformer, the sum of the magnetomotive forces in a given direction around the core caused by currents in the three windings is zero, just as in the two-winding transformer. Thus, for the transformer in Fig. 32,

$$i_bN_1 - i_b'N_1 - i_2N_2 = 0, \tag{208}$$

or

$$i_2 = \frac{N_1}{N_2}(i_b - i_b'). \tag{209}$$

A rigorous analysis of the push-pull amplifier requires a consideration[14] of the finite magnetizing impedance and also of the leakage reactances among the three windings of the output transformer, but their effects are neglected here.

Substitution of Eqs. 205 and 207 in Eq. 209 gives the current in the load as

$$i_2 = 2\frac{N_1}{N_2}\sqrt{2}(I_{p1}\cos\omega t + I_{p3}\cos 3\omega t + \cdots). \qquad [210]$$

Thus the effect of the symmetrical arrangement is to cause a cancellation of the average components, the second harmonics, and all other even harmonics generated within the tube. However, if the ***grid-signal voltage is nonsinusoidal,*** all frequencies present in it, including even harmonics, are amplified as usual. The absence from the output signal of components resulting from even-harmonic generation is one of the advantages of the push-pull connection over the parallel connection of tubes.

The current through the plate-power supply or battery is the sum of the currents through the two tubes. From Eqs. 205 and 207, this is

Current through plate-power
supply $= i_b + i_b'$

$$= 2I_{b0} + 2I_{p0} + \sqrt{2}(2I_{p2}\cos 2\omega t + 2I_{p4}\cos 4\omega t + \cdots). \quad [211]$$

The average value of this plate-power-supply current as indicated by a direct-current ammeter increases by the amount $2I_{p0}$ when the grid-signal voltage is increased from zero to E_g. A change in the ammeter indication when the grid-signal voltage is applied therefore indicates waveform distortion caused by the generation of harmonics. Whereas only the odd harmonic-generation components exist in the output current and voltage, the current through the plate-power supply contains only the even harmonic-generation components.

Waveforms of the grid-signal voltages, plate currents, and output current, illustrating the operation when the path of the operating point extends into the nonlinear region of the tube characteristics, are shown in Fig. 34. The grid-signal voltages e_g and e_g' are sinusoidal and 180 degrees out of phase. The plate-current waveforms i_b and i_b' are flattened at the bottom because of nonlinearity of the tube characteristics, but each is a replica of the other displaced by 180 degrees. The output current i_2 also is flattened near its crests, and, since the diagram shows that

$$i_2(\omega t) = -i_2(\omega t + \pi), \qquad [212]$$

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[14] A. P-T. Sah, "Quasi Transients in Class B Audio-Frequency Push-Pull Amplifiers," *I.R.E. Proc.*, *24* (1936), 1522–1541.

i_2 contains only odd harmonics,[15] as was deduced analytically in Eq. 210.

A graphical analysis for the path of operation on the plate characteristics over an extended range is not readily made for an individual tube, because of the coupling between the plate circuits of the two tubes through the output transformer. However, as is shown subsequently, the operation of the circuit can be represented graphically by construction of the plate characteristics for a *composite tube*, the composite tube being defined as one which, operating into one-half the output transformer primary winding with the other half open-circuited, gives the same current and power in the load as the two tubes in push-pull. The path of operation on these *composite characteristics* is a straight line, and the methods of finding the power output and harmonic generation given in Art. 8 are applicable. It is assumed again in this analysis that the output transformer is ideal, thus having no resistance, leakage reactance, exciting current, or losses, and that the load is a pure resistance. The circuit diagram is again that of Fig. 30, and Eq. 209 applies to the plate circuit.

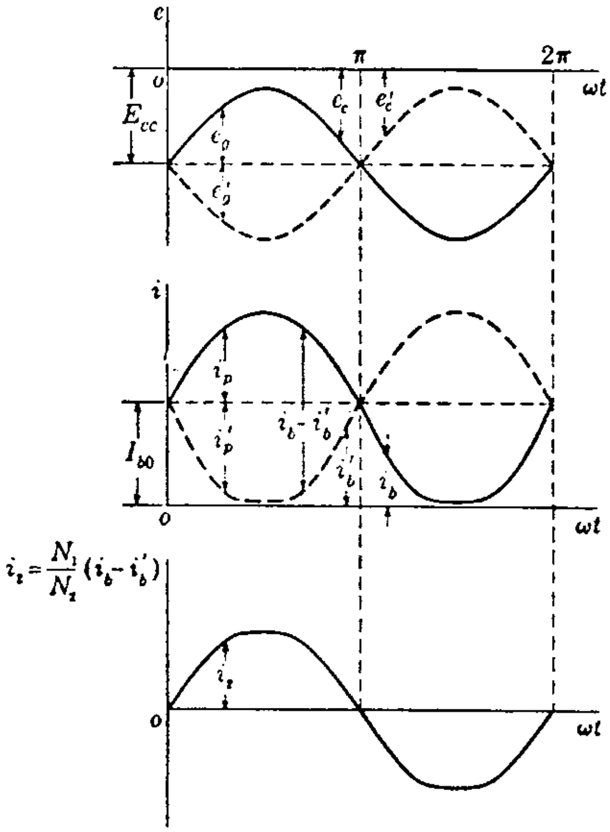

FIG. 34. Waveforms in a push-pull amplifier when the operation extends into the nonlinear region of the tube characteristics.

Equation 209 shows that the output current i_2 is the same as the output current that would exist if an equivalent current

$$i_d = i_b - i'_b \qquad [213]$$

existed in one-half the transformer primary winding. Thus the operation of the circuit in Fig. 30 is the same as that in Fig. 35, where

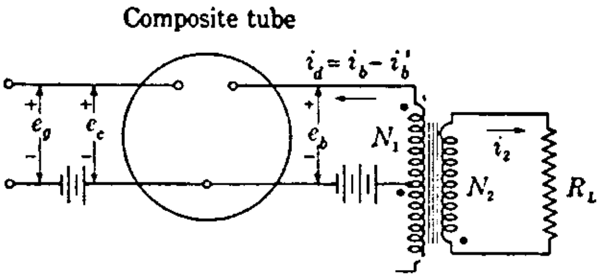

FIG. 35. Composite tube and circuit equivalent to that of Fig. 30.

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[15] P. Franklin, *Differential Equations for Electrical Engineers* (New York: John Wiley & Sons, 1933), 65.

the composite tube has a plate current i_d and the relationships among i_d, e_b, and e_c are yet to be found.

In functional notation, Eq. 213 becomes

$$i_d(e_c, e_b) = i_b(e_c, e_b) - i_b'(e_c', e_b'), \qquad [214]$$

where the two terms on the right-hand side of the equation represent the characteristics of the individual tubes. These can be combined in accordance with the circuit restrictions as follows: Since the input transformer is ideal,

$$e_g = -e_g'; \qquad [215]$$

thus

$$e_c' = E_{cc} + e_g' = E_{cc} - e_g = e_c - 2e_g. \qquad [216]$$

Also, since the ideal output transformer acts as an autotransformer between the two plate circuits, it makes

$$e_p = -e_p'. \qquad \blacktriangleright[217]$$

With two separate load resistors substituted for the transformer and load, Eq. 217 would not be correct, and *the entire analysis that follows is therefore true only when the transformer is used.* Equation 199, which was obtained in the linear analysis, does not apply to the nonlinear operation of the tube.

Since the resistance of the transformer is negligible,

$$E_{b0} = E_{b0}' = E_{bb}; \qquad [218]$$

thus

$$e_b' = E_{bb} + e_p' = E_{bb} - e_p \qquad [219]$$

and

$$e_b = E_{bb} + e_p, \qquad [220]$$

whence

$$e_b' = 2E_{bb} - e_b. \qquad [221]$$

Since the tubes are assumed to be identical, the function i_b is the same form as the function i_b', but they are functions of different variables; thus the prime may be dropped if the variables are indicated, and substitution of Eqs. 216 and 221 in Eq. 214 gives

$$i_d(E_{cc} + e_g, e_b) = i_b(E_{cc} + e_g, e_b) - i_b(E_{cc} - e_g, 2E_{bb} - e_b). \qquad [222]$$

In this way, once the power-supply voltages E_{bb} and E_{cc} are selected, the independent variables are reduced to two, namely, e_g and e_b, and the characteristics may be graphically constructed as shown in Fig. 36, where the curves for zero grid-signal voltage e_g are drawn. The plate-current characteristic curve for the composite tube is obtained through rotating the plate characteristic for an individual tube through 180 degrees about the origin, then displacing the curve along the axis of abscissas until its new origin falls at the point at which e_b is equal to $2E_{bb}$ on the original scale, and, finally, subtracting the magnitudes of the ordinates of the two curves. Thus the length of the ordinate $\overline{xy}$ equals $\overline{xz}$ minus $\overline{xv}$ in Fig. 36. Several features of the composite characteristics are at once apparent; namely, (a) the quiescent plate current in the composite tube is zero; (b) the composite plate characteristic is much straighter than either of the individual tube characteristics, although it may have more curvature than that shown in Fig. 36; (c) the plate resistance of the composite tube $\partial e_b/\partial i_d$ is one-half the plate resistance of either of the individual tubes at the quiescent operating point; and (d) the plate resistance for the composite tube is essentially constant over the range shown, though the plate resistances of the individual tubes vary considerably.

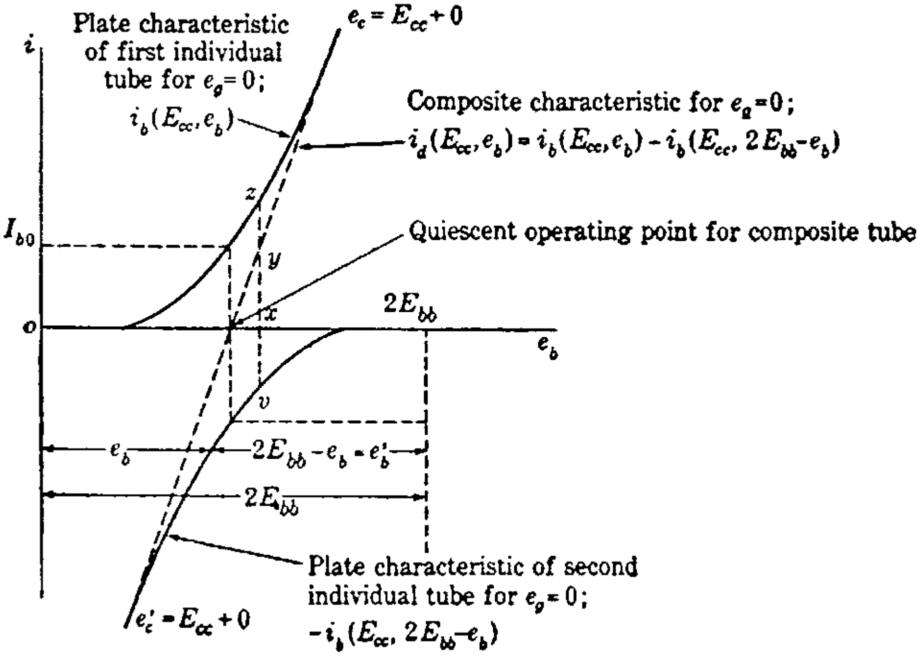

FIG. 36. Construction of a plate characteristic curve for the composite tube with zero grid-signal voltage.

The construction of the plate characteristics of the composite tube corresponding to three particular values of grid-signal voltage e_g is shown in Fig. 37. One particular value of the grid-signal voltage is zero, another is positive, and the third is equal in magnitude to the positive one but is negative. The curves for zero grid-signal voltage lie between the others in the figure and are similar to those in Fig. 36. To obtain the composite characteristic curve for the particular positive value of grid-signal voltage, the curve for an equal negative grid-signal voltage, denoted by $i_b(E_{cc} - e_g, e_b)$ on the diagram, is rotated and displaced as previously explained, whereupon it becomes the curve denoted by

$$-i_b(E_{cc} - e_g, 2E_{bb} - e_b).$$

The ordinates of this latter curve are then added algebraically to those of the curve for the particular positive value of grid-signal voltage, denoted by $i_b(E_{cc} + e_g, e_b)$, giving the left-hand dotted line with positive slope. Again, on this line, the length of the ordinate $\overline{xv}$ is subtracted from that of the ordinate $\overline{xz}$ to give the length $\overline{xy}$, and the resulting curve is the characteristic of the composite tube for the particular positive value of e_g. The construction of the right-hand dotted line, which is the characteristic of the composite tube for the particular negative value of e_g, is done in a similar manner. Note that composite characteristics for equal positive and negative grid-signal-voltage increments are images of each other about the quiescent operating point. This symmetry is the reason for several important operating features of the push-pull circuit with an output transformer.

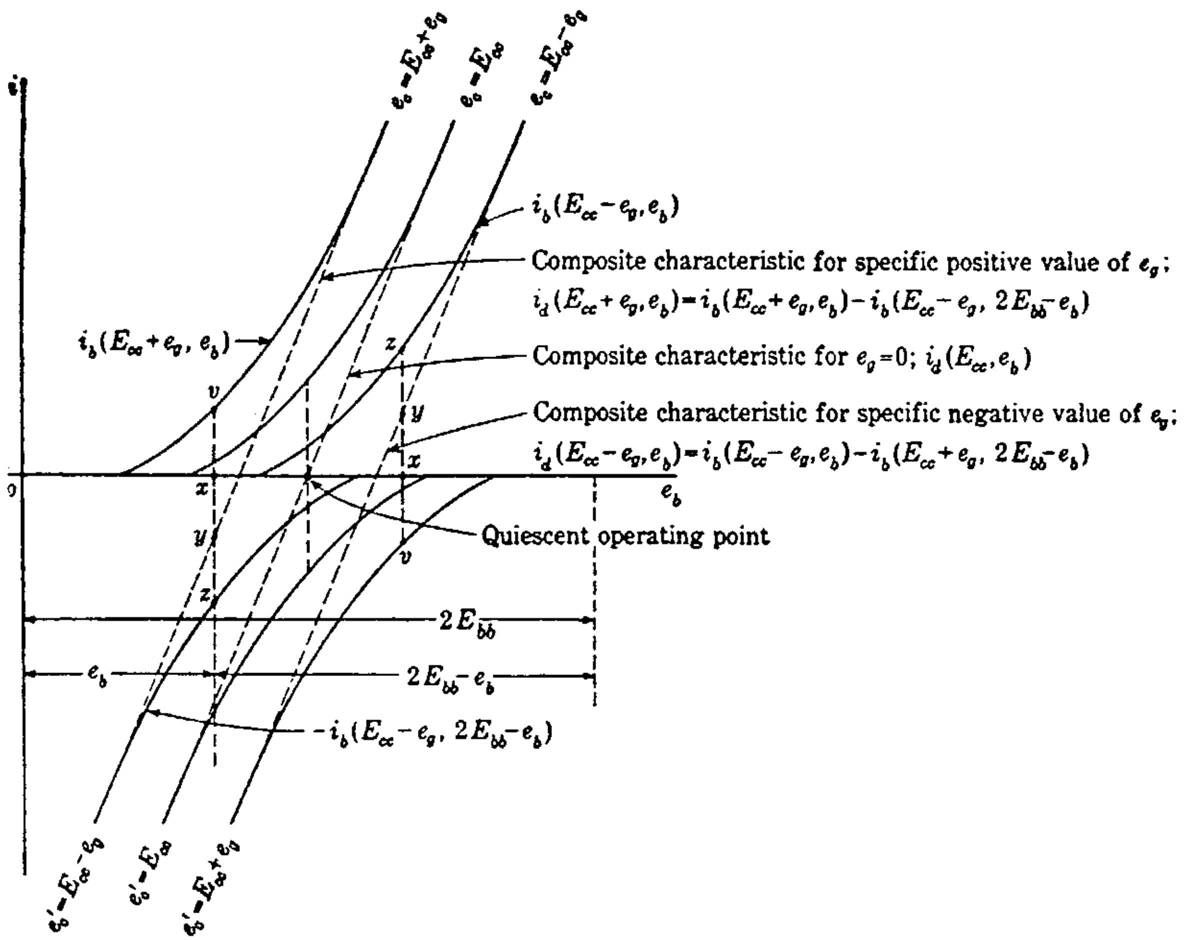

FIG. 37. Construction of the plate characteristics of the composite tube for three specific values of grid-signal voltage.

Figure 38 shows a family of composite characteristics for two Type 45 tubes in push-pull with a plate-supply voltage of 240 volts. The characteristics therefore spread over a range of 480 volts. The grid-bias voltage is -50 volts, and the heavy dotted lines with positive slopes are the two individual tube characteristics for zero grid signal voltage. When the ordinates of these curves are added algebraically, the result is the heavy solid line with positive slope, which is the composite characteristic for

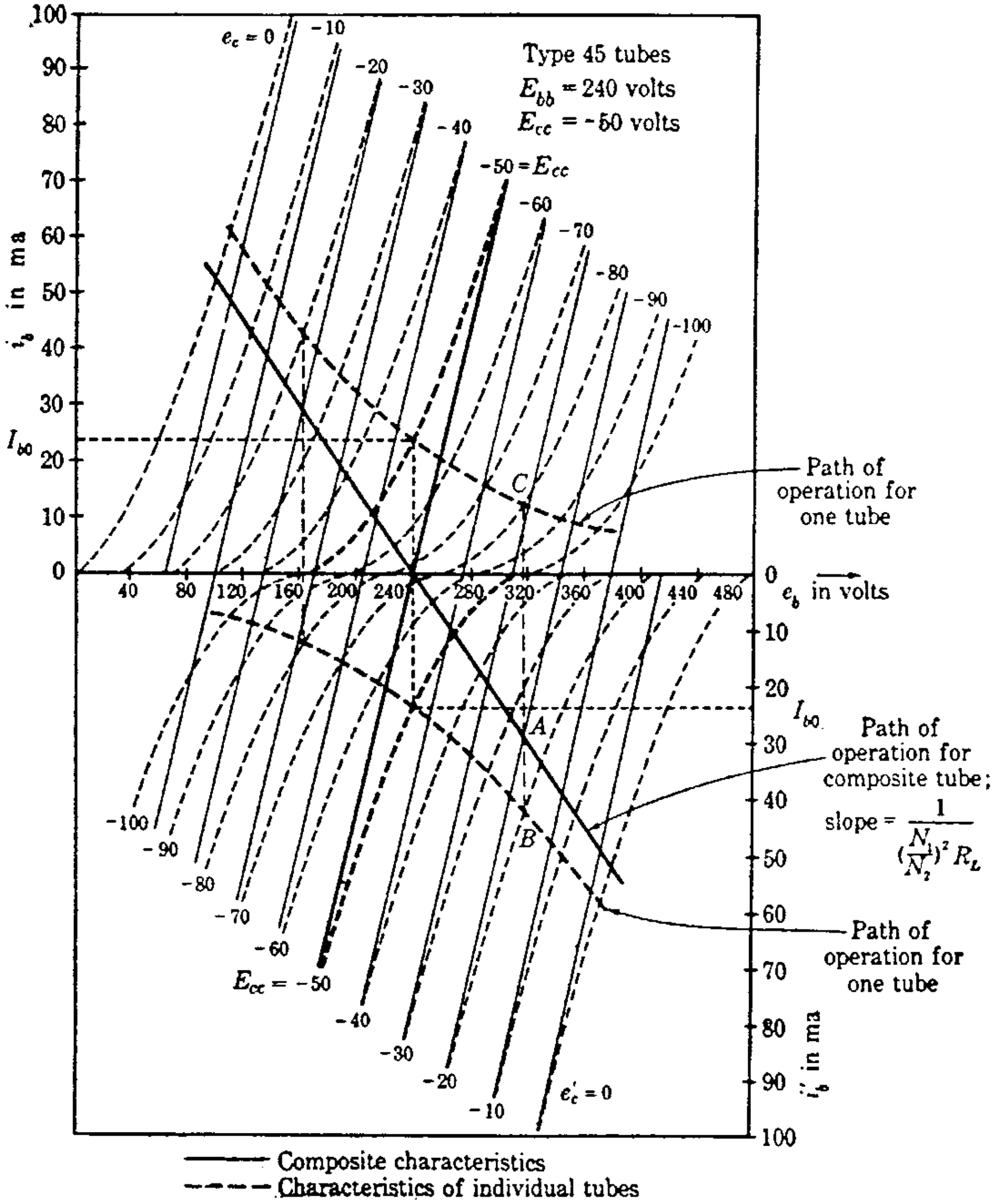

FIG. 38. Composite characteristics for two Type 45 tubes in a push-pull circuit at specific values of grid-bias voltage and quiescent plate voltage.

zero grid-signal voltages. The light solid lines with positive slopes are the composite characteristics for 10-volt increments of grid voltage constructed by the method shown in Fig. 37. The solid and dotted lines with negative slope are discussed subsequently.

Not only the characteristic corresponding to the zero grid-signal-voltage condition but all the composite characteristics over a wide range of grid voltage are essentially straight lines. The grid-bias voltage E_{cc}, chosen in Fig. 38 as -50 volts, is one for Class A operation. In later discussions of Class AB and Class B push-pull operation, it is shown that the composite characteristics are not always straight for those operating conditions. Note that the characteristics of the composite tube as defined here are dependent upon the values of E_{cc} and E_{bb} and thus depend on quantities external to the tube. The composite tube differs from an ordinary tube in this respect.

The composite plate characteristics from Fig. 38 are reproduced in Fig. 39. As far as the current, voltage, and power in the load resistor are concerned, the operation of the push-pull circuit is equivalent to that of Fig. 35, where the characteristics of the composite tube are given by Fig. 39. The quiescent operating point is on the abscissa axis at e_b equals E_{bb}, where i_d equals zero, and a load line having a slope $-\dfrac{1}{\left(\dfrac{N_1}{N_2}\right)^2 R_L}$ gives the path of operation on the characteristics, as is shown by the solid line of negative slope in Figs. 38 and 39. The waveform distortion that occurs because of harmonic generation in the tubes may be obtained from this load line by the methods of Art. 8, if i_b is replaced by i_d, and negative values of i_d are recognized. Because of the symmetry of the composite characteristics mentioned previously, no steady or even-harmonic components appear in i_d or in the load current.

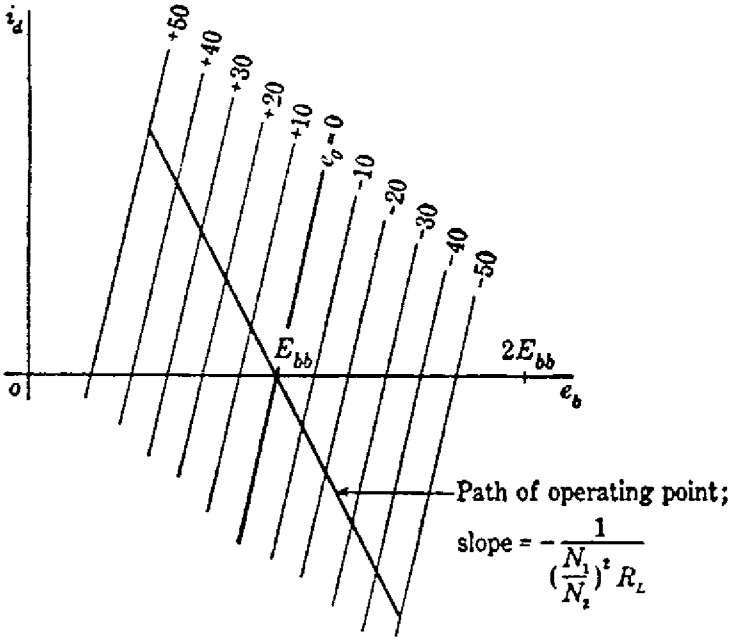

FIG. 39. Plate characteristics of the composite tube with load line superposed.

Although the plate currents in the individual tubes may decrease to zero and remain there for an appreciable fraction of a half-cycle, such behavior is not apparent on the composite plate characteristics, since the operation along the load line is entirely symmetrical about the quiescent operating point for positive or negative values of grid-signal voltage. However, the paths of operation for the individual tubes can be found readily by a process which is the reverse of the one by which the composite characteristics are constructed. Thus, in Fig. 38, a vertical line through the intersection at *A* of the path of operation and a particular composite characteristic intersects at *B* and *C* the two individual tube characteristics from which the composite characteristic is constructed, thereby disclosing the individual tube plate currents for a particular value of grid-signal voltage. By this method, the paths of operation for the individual tubes shown by the dotted curves with negative slopes are constructed. They are curved, even though the load is purely resistive and the output transformer is ideal.

For the particular conditions illustrated in Fig. 38, the individual tube plate currents do not fall to zero when the range of operation is limited by the two curves corresponding to zero grid voltage on the two tubes; thus the operation is Class A_1. However, if the grid-bias voltage is chosen somewhat larger, the individual tube currents may be zero for an appreciable fraction of the cycle, and operation changes to Class AB_1, which is discussed in more detail in Ch. X. Figure 40 shows a limiting example for Class A_1 operation, since the individual plate currents in it just reach zero when the grid voltage of the opposite tube reaches zero.

The considerations that govern the maximum power output with a prescribed amount of harmonic generation from a push-pull amplifier are quite different from those for a single-tube amplifier given in Art. 11. Since the even harmonics generated in the tubes are canceled in the output transformer, the total generation of harmonics in the amplifier is smaller in the push-pull amplifier than in the single-tube amplifier when the tubes have the same operating voltages and deliver the same power output individually. Consequently it follows that, for the same total harmonic generation in the amplifier, the maximum power output from each tube is larger in the push-pull amplifier than in the single-tube amplifier. The increase may be as much as 50 per cent. This increased power output is made possible by the fact that changes both in the operating voltages and in the load resistance effective in the plate circuits of the individual tubes may be made under the specified conditions. Since the even harmonics are canceled, the path of operation may be extended farther into the lower region of the tube characteristics in a push-pull amplifier than is indicated in Fig. 26, Art. 11, for a single-tube amplifier when a prescribed amount of harmonic generation is not to be exceeded. Accordingly, for the same amount of harmonic generation, a larger magnitude of grid-bias voltage and a larger grid-signal voltage amplitude may be used in

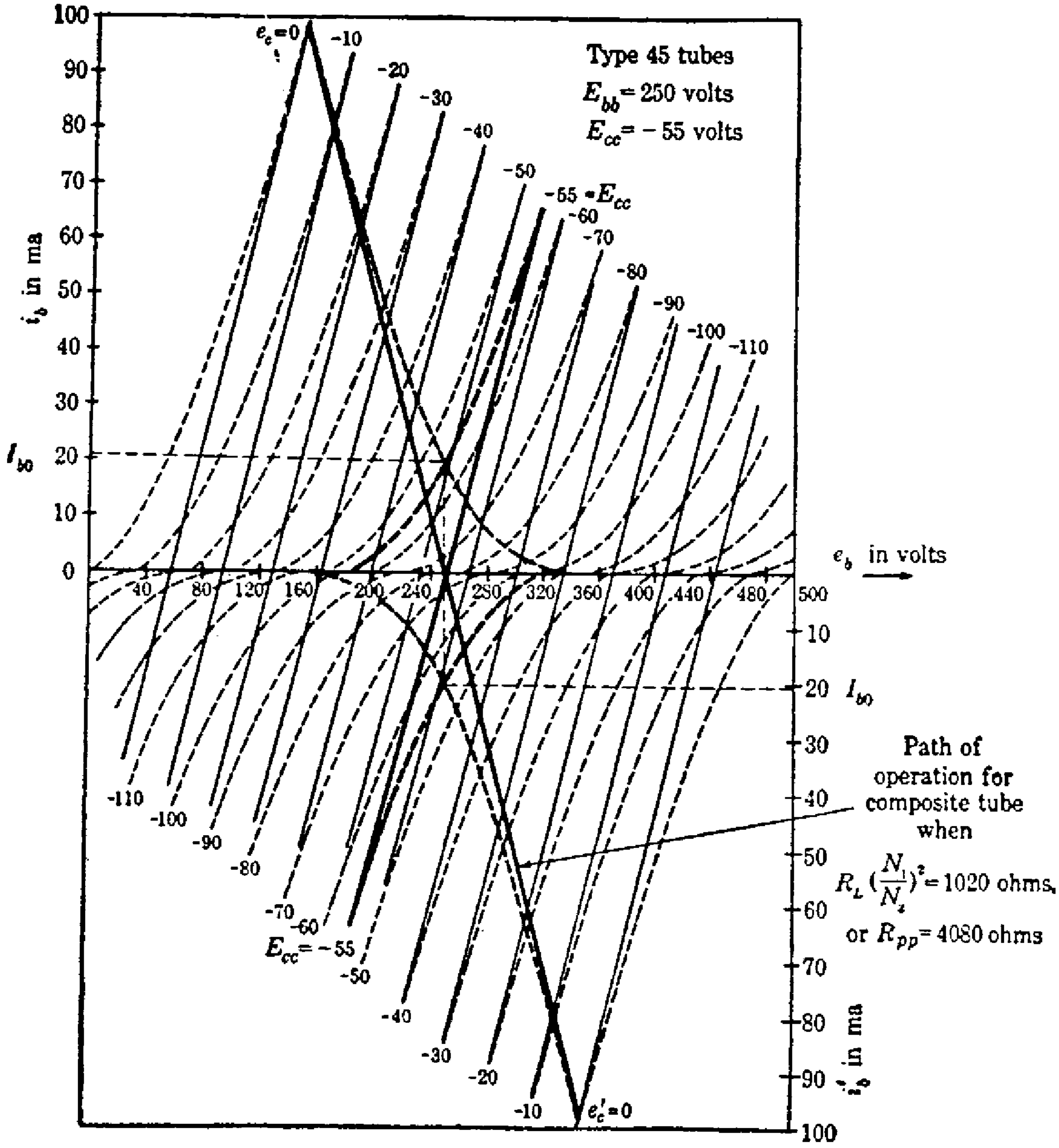

FIG. 40. Composite characteristics for two Type 45 tubes in a push-pull circuit for the limiting condition of Class A_1 operation.[*]

the push-pull amplifier than in the single-tube amplifier, and the power output from each tube is therefore larger.

The change of voltages described in the preceding paragraph is one factor that contributes to the increased value of maximum power output in the push-pull amplifier. Another factor is the change that may be made in the effective load resistance for each tube. Since the composite charac-

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[*] This diagram is adapted from B. J. Thompson, "Graphical Determination of Performance of Push-Pull Audio Amplifiers," *I.R.E. Proc.*, *21* (1933), Fig. 8, p. 595, with permission.

teristics are symmetrical about the curve for zero grid-signal voltage, and are practically straight and parallel for Class A_1 operation, the amount of harmonic generation is essentially independent of the plate-to-plate load resistance for a particular value of grid-signal-voltage amplitude. Consequently, the considerations of Art. 11 and the result that the effective load resistance for a tube must be approximately equal to twice the plate resistance of the tube for maximum power output with a prescribed amount of harmonic generation and a prescribed quiescent plate voltage do not apply to the composite tube. Instead, the considerations of Art. 10 apply, and the slope of the path of operation on the composite characteristics for maximum power output is equal to the negative of the slope of the composite characteristics. The slope of the path of operation on the composite characteristics corresponds to $(N_1/N_2)^2R_L$, which is one-fourth the plate-to-plate resistance given by $4(N_1/N_2)^2R_L$. Thus the plate-to-plate resistance should equal four times the plate resistance of the composite tube. The plate resistance of the composite tube is, however, approximately one-half the plate resistance of the individual tubes at the quiescent operating point. The optimum plate-to-plate resistance for the push-pull amplifier therefore is twice the plate resistance of the individual tubes, and the optimum value of the load resistance effective in the plate circuit of each tube hence is equal to the plate resistance of the tube. In accordance with Eq. 177, this condition results in an increase of power output from each tube over the value obtained when an effective load resistance equal to twice the plate resistance of the tube is used.

The discussion in this article applies to triodes used with an output transformer for delivering power to a resistance load. The use of the push-pull connection as a balanced voltage amplifier without an output transformer is discussed in Ch. IX and Class AB, Class B, and Class C operation is discussed in Ch. X.

15. Symbols for vacuum-tube circuit analysis

In the preceding articles of this chapter a number of special symbols are introduced and defined. A large number of symbols are needed in analysis of vacuum-tube circuits, because the operation is complicated by the superposition of direct quantities and alternating quantities having several harmonic components. Confusion is likely to result if a consistent set of symbols is not defined and adhered to through all the analysis. The methods of circuit analysis given in the volume on electric circuits are directly applicable to vacuum-tube circuits and may be used with arbitrarily assigned positive directions of currents and voltages. The consistent set of definitions adopted in this volume is merely one of the innumerable possible sets. It is adopted because it is in substantial agreement with the latest standards[16] available; thus it is the set most likely to be encountered by the reader in other publications in the future. To eliminate one additional source of confusion, the definitions here are extended to include the assigned positive *direction* of the quantities. If symbols defined for the directions opposite to those chosen here are needed, other symbols can be used.

Table I summarizes the symbols for the electrical parameters of the tube and circuit and for some of the currents and voltages that do not enter into most of the problems. Table II gives the definitions and symbols of the current and voltage components that are fundamental to the operation of triode circuits. In Table II, the first four rows contain symbols pertaining to the total quantities, and the fifth, sixth, and seventh rows, symbols which pertain to the varying components and are useful in circuit analysis when harmonic generation is neglected. The last four rows contain symbols useful in representing nonsinusoidal varying components as a Fourier series. Complex quantities are indicated by roman type.

TABLE I

SYMBOLS FOR VACUUM-TRIODE CIRCUITS

g_p = plate conductance
r_p = plate resistance
g_g = grid conductance
r_g = grid resistance
g_m = grid-plate transconductance (mutual conductance)
μ = amplification factor $= -\left.\dfrac{de_b}{de_c}\right|_{i_b\ constant}$
C_{gp} = grid-plate capacitance
C_{gk} = grid-cathode capacitance
C_{pk} = plate-cathode capacitance
E_{bb} = plate-supply voltage rise from the cathode toward the plate
E_{cc} or E_{cc1} = control-grid supply voltage rise from the cathode toward the grid
E_{cc2} = screen-grid supply voltage rise from the cathode toward the screen grid
E_{ff} = filament or heater supply voltage (effective or direct value)
E_f = filament or heater terminal voltage (effective or direct value)
I_f = filament or heater current (effective or direct value)
I_s = total electron-emission (saturation) current from the cathode

Tubes with more than one grid require additional symbols which are supplied as follows[16]:

"*Generalized System for Multigrid Tubes.* The following scheme of symbols for multigrid tubes avoids the extension of letter subscripts and provides a framework of symbols for tubes with any number of grids. In this system the grids are numbered according to position, the grid immediately adjacent to the cathode or filament being No. 1, the next grid No. 2,

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[16] *Standards on Electronics* (New York: The Institute of Radio Engineers, 1938), 11–14.

TABLE II. SYMBOLS FOR TRIODE CIRCUITS

Component	*Name and Direction of Quantity*				
	Voltage rise from cathode to grid	*Voltage rise from cathode to plate*	*Current through the external circuit toward the grid*	*Current through the external circuit toward the plate*	*Voltage drop across the load in the direction of positive plate current*
Instantaneous total value	e_c	e_b	i_c	i_b	e_L
Quiescent value; steady value when varying component of grid voltage is zero	E_{c0}	E_{b0}	I_{c0}	I_{b0}	E_{L0}
Average value of the total quantity	E_c	E_b	I_c	I_b	E_L
Instantaneous maximum of the total quantity	E_{cm}	E_{bm}	I_{cm}	I_{bm}	E_{Lm}
Instantaneous value of the varying component	e_g	e_p	i_g	i_p	e_z
Effective value of the varying component	E_g	E_p	I_g	I_p	E_z
Amplitude of the varying component	E_{gm}	E_{pm}	I_{gm}	I_{pm}	E_{zm}
Average value of the varying component	E_{g0}	E_{p0}	I_{g0}	I_{p0}	E_{z0}
Instantaneous value of the harmonic components	$e_{g1}, e_{g2}, \cdots$	$e_{p1}, e_{p2}, \cdots$	$i_{g1}, i_{g2}, \cdots$	$i_{p1}, i_{p2}, \cdots$	$e_{z1}, e_{z2}, \cdots$
Effective value of the harmonic components	$E_{g1}, E_{g2}, \cdots$	$E_{p1}, E_{p2}, \cdots$	$I_{g1}, I_{g2}, \cdots$	$I_{p1}, I_{p2}, \cdots$	$E_{z1}, E_{z2}, \cdots$
Amplitude of the harmonic components	$E_{g1m}, E_{g2m}, \cdots$	$E_{p1m}, E_{p2m}, \cdots$	$I_{g1m}, I_{g2m}, \cdots$	$I_{p1m}, I_{p2m}, \cdots$	$E_{z1m}, E_{z2m}, \cdots$

etc. In designating the voltages or currents associated with a particular grid, the symbols given on the preceding pages will be used with the grid number as a subscript. . . . Control-grid symbols are frequently used where reference is not made to other grids. The number of the grid need not be used in this case. It will be understood that, when no number appears in the subscript, the reference is to the control grid."

It should be noted that one possible source of confusion lies in the fact that some of the symbols in the last three rows of Table II are also used with a different meaning for multigrid tubes. However, this does not lead to difficulty in any of the problems treated in this text.

PROBLEMS

1. A triode having the plate characteristics of Fig. 7, Ch. IV, is used with a plate-supply voltage E_{bb} of 400 volts, a load resistance R_L of 100,000 ohms, and a grid-bias voltage E_{cc} of -3 volts. What is the quiescent plate current I_{b0}?

2. A relay having a resistance of 1,000 ohms is to be operated by the plate current of a high-vacuum triode. If the available direct grid-signal voltage is 5 volts and the relay closes at 30 ma and opens at 20 ma, which of the triodes whose plate characteristics appear in some one manufacturer's literature should be satisfactory? For each triode selected, specify the plate-supply and grid-bias voltages that must be used.

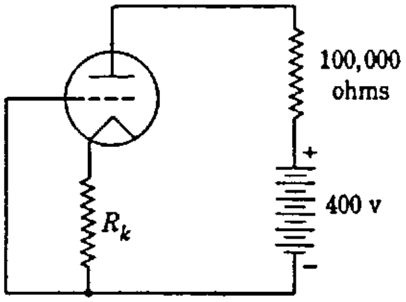

FIG. 41. Triode circuit for Prob. 3.

3. A triode has the plate characteristics given in Fig. 7, Ch. IV, except that the grid-voltage scale is to be multiplied by ten — that is, the increment in grid voltage between adjacent curves is 10 volts instead of 1 volt. The tube is connected as shown in Fig. 41 with a 400-volt battery as a plate-power supply and a plate-load resistance of 100,000 ohms. The resistor R_k is so adjusted that there is a voltage of 50 volts between the grid and the cathode.

Find the quiescent plate current I_{b0}, the quiescent plate voltage E_{b0}, and the required value of R_k.

4. The plate current of a particular triode is satisfactorily given by the expression

$$i_b = 17 \times 10^{-5}\left(e_c + \frac{e_b}{8}\right)^{1.7} \text{ amp,}$$

where e_c and e_b are in volts.

(a) Determine the plate current i_b corresponding to a grid voltage e_c of -15 volts and a plate voltage e_b of 200 volts.

(b) Find the dynamic plate resistance r_p and the mutual conductance g_m of the tube at the operating point specified in (a).

(c) If the tube is used as a Class A_1 voltage amplifier with a load resistance of 10,000 ohms and a grid-bias voltage E_{cc} of -15 volts, what plate-supply voltage E_{bb} is required to produce a quiescent plate current equal to that determined in (a)?

(d) Determine the voltage gain of the amplifier for the conditions in (c).